1 Quiz 4 Problem 1 (7.5.36)

Evaluate $\int \frac{1}{x^2+4x+7}dx$.

We will complete the square, do a U-sub then use the formula $\frac{1}{a}\arctan(\frac{x}{a}) = \int \frac{dx}{x^2+a^2}dx$.

To complete the square on the bottom, add and subtract $(\frac{b}{2a})^2 = (4/2)^2 = 4 : \frac{1}{x^2+4x+7} = \frac{1}{x^2+4x+7\pm 4} = \frac{1}{(x^2+4x+4)+7-4} = \frac{1}{(x+2)^2+3}$.

Now, we'll set $u = x+2$, so $du = dx$. Then,

$$\int \frac{dx}{x^2+4x+7} = \int \frac{dx}{(x+2)^2+3} = \int \frac{du}{u^2+(\sqrt{3})^2} = \frac{1}{\sqrt{3}}\arctan(\frac{u}{\sqrt{3}}) \tag{1}$$

$$= \frac{\sqrt{3}}{3}\arctan(\frac{\sqrt{3}(x+3)}{3}) + C \tag{2}$$

2 OR Quiz 4 Problem 2 (7.5.53)

Evaluate $\int_0^2 \frac{1}{\sqrt{16-x^2}}dx$.

We use the formula $\arcsin(\frac{x}{a}) = \int \frac{dx}{\sqrt{a^2-x^2}}$ with $a = 4$:

$$\int_0^2 \frac{1}{\sqrt{16-x^2}}dx = \int_0^2 \frac{1}{\sqrt{4^2-x^2}}dx = [\arcsin(x/4)]_0^2 = \arcsin(2/4) - \arcsin(0) = \arcsin(1/2) \tag{3}$$

$$= \pi/6 \tag{4}$$

3 OR Quiz 4 Problem 3

Show that $\lim_{x\to 0^+} x^{(x^x)} = 0$.

Well,

$$\lim_{x\to 0^+} x^{(x^x)} = \lim_{x\to 0^+} e^{\ln(x^{(x^x)})} \tag{5}$$

$$= \lim_{x\to 0^+} e^{x^x \cdot \ln(x)} \tag{6}$$

$$= e^{\lim_{x\to 0^+} x^x \cdot \ln(x)} \tag{7}$$

by the continuity of the function e^x since limits commute with continuous functions, i.e if f is continuous, and g is any function, $\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$.

Now, using the fact that $\lim_{x\to 0^+} x^x = 1$ and $\lim_{x\to 0^+} \ln x = -\infty$, we have that $e^{\lim_{x\to 0^+} x^x \cdot \ln(x)} = e^{-\infty} = 0$.